Quality, Quantity, Granularity, and Thresholds of Emergence

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Abstract: Hayek's distinction between *taxis*—what is controlled or managed—and *cosmos*—what spontaneously organizes itself—corresponds to two kinds of order in the world, the predictable and the emergent, the quantitative and the qualitative. A controller must know the outcome of his actions in advance, and such predictions must be based on probabilistic analysis based in turn upon quantitative and continuous differences in the world. Not all changes and differences in the world are so; spontaneous emergence creates qualitatively new forms of order with new rules not to be deduced from the old. Yet quantitative changes can cross a threshold and suddenly amount to qualitative ones. Critical mass occurs with a sufficiently rich, complex, recursive and exponentially increasing body of feedback among the granular elements of a system, whose mathematical difficulty requires new kinds of solutions. For such a body of feedback to arise, relatively autonomous individualities or "counters" with different strategies must compete with each other in ways described by game theory. The paper illustrates several such individualities and the results of their interaction in the inanimate, living, and social worlds.

Keywords: "Amounts to"; analog; catastrophe; cosmos; creativity; critical mass; difficulty; digital, dynamical systems; emergence; feedback; freedom; games; granularity; Hayek; individuality; Polanyi; probability; prediction; Prigogine; quality; quantity; Shannon; spontaneous order; Taleb; taxis; thresholds; time.

Quality and quantity have long been thought by philosophers to be separate universes. Since the time of Newton and Laplace, who presented a view of the physical universe that was materialist, reductionist, and determinist, the only way that philosophy could rescue the concept of freedom and with it the world of value, responsibility, meaning, and beauty was to export it from the world of matter and energy into some other realm—and hence the division in our universities between the sciences and the humanities, *Naturwissenschaft* and *Geisteswissenschaft*.

But the emergence of new order with qualitatively unprecedented characteristics occasioned by the quantitative crossing of natural thresholds questions any simple dualism. The presence of natural thresholds requires that the universe cannot be understood as composed only of smooth gradients analyzable by probabilistic and statistical means. "Catastrophes" and "cusps" in René Thom's terminology—the appearance of radical breaks and discontinuities—are as natural as continuities and predictable outcomes. Nobody (except, by coincidence, habitual prophets of doom) predicted the recent banking and mortgage crisis; nobody could have predicted the evolution of the human species; I cannot predict what insight I will come to in the course of composing this essay.

Certainly many events can be predicted. We can predict planetary eclipses; the interference pattern formed by a sufficiently large number of photons fired from a source through a double slit; the pressure of a given quantity of a given gas confined in a given volume at a given temperature; the effect of an increase in the money supply on inflation; even the effect of a well-known drug on the brain. Such events are controllable by individual conscious design and thus potentially reducible to what Hayek calls *taxis*. But they do not by any means encompass events that are either unpredictable by a system less complex than they are, such as a human controller, or constitutively unpredictable in themselves. Nonlinear dynamical systems, as Ilya Prigogine famously observed, are the most salient sources of such unpredictability, as the effect of their feedbacks collapses from the homeostatic to the chaotic. But it is in the regime of the chaotic that new orders emerge, as locally stable equilibria.

The argument of this essay is that a *cosmos* depends on the presence of nonlinear dynamical systems at any level from the subatomic to the sociological, and that such systems both originate from and generate multiple and locally stable equilibria. Cosmic reality comes in semi-autonomous clumps, which can become the pixels of bigger and quite unexpected pictures, or the nodes of larger and previously unimagined networks, according to emergent orders that become possible once those relatively stable clumps are established. The clumping of quarks gives rise to a new language of subatomic particles which, when certain boundary conditions of temperature and pressure are crossed, can clump to make atoms and molecules, granular individuals that can in turn clump to make crystals and living organisms. The gradients by which such boundary conditions are approached can be smooth and quantitative, but the crossings are abrupt and qualitative. In other words, the natural and human universe is not only smooth and predictable but granular. It is quantized like the counters and turns of a game, and capable of unpredictable creativity: it constitutes Hayek's notion of cosmos. The boundary or threshold between the quantitative and the qualitative, the smooth gradient and the cusp, the homogeneous mass and the clump, is approached when the difficulty of calculating the future of a system increases exponentially, or in Michael Polanyi's terms becomes massively unmanageable (Polanyi, 1951).

This increase in difficulty happens when infinite feedback loops appear within a collection of granular participants—in other words, in a game-like system. Mathematics implies a threshold between the computable and the incomputable—calculation is itself a controlled game using discrete turns and counters, but it ends up positing infinitely "hard" solutions. Physics requires quanta and a distinction between the reversible and the irreversible. Chemistry dictates catastrophic changes of state. Evolutionary biology requires distinct competing entities at every level of organization from the gene and cell up to the individual and the species. Neuroscience requires the emergence of qualitatively irreducible capacities such as perception and cognition. Information theory posits a distinct boundary between information and noise. Economic activity implies unpredictable financial crises and game-changing innovations. Time itself may be related to the threshold between the computable and the incomputable.

Freedom and responsibility, art and poetry can be understood in the light of threshold-crossing, and may indeed be traditional concepts and capacities that are our closest approach to anticipating threshold-crossings and understanding emergent spontaneous orders. The implications for the social sciences and for public policy, though they confirm the insights of classical liberal thought about the limits of human planning, also suggest further perspectives and rich possibilities for the future.

"THRESHOLDINESS"

What Newton and Laplace, the heroes of quantitative science, could not have known was something that has only recently become clear: that a quantitative difference, if close enough to some great natural threshold or inherent and constitutive instability in the world, can trigger a qualitative difference. Today's market risk analysts, as Nassim Nicholas Taleb has pointed out in his book *The Black Swan* (Taleb, 2010), are no better at detecting the imminence of such threshold-crossings, such inherent instability, than was the founder of probability theory and statistics, Blaise Pascal. The recent banking crisis was the result. Mark Twain's famous aphorism, quoted by Churchill—"there are lies, damn lies, and statistics"—has been proven true yet again: the emergent whole is so often greater than the sum of its parts, and statistics can only deal with parts.

That instability, that "thresholdiness"—the demon that haunts all worldly calculation, because of its unpredictability—has been explored by Thom (1994), Mandelbrot (1982), Lestienne (1998), and Prigogine (1997). The instability is perhaps just as interesting as the nature of "qualitativeness" that has bedeviled philosophers for so many millennia. It is the threshold between the quantitative and the qualitative *in itself*, the way that "something can amount to something." In Mandelbrot's insight, a Peano space-filling curve,[1] which is after all only a line, can "amount to" a plane, if a plane is defined as a two-dimensional space in which all the locations are occupied. A frilly crocheted plane, a flower whose bell results from more growth of cells per open unit of space than there is space for on the plane, can "amount to" a negative curvature and thus traces out a volume. Seven (but not five) H_2O molecules "amount to" water, with its constitutive wetness, flow, surface tension, ripples, bubbles, capillary action, drops, meniscus, and so on. It is only at the threshold of six that there emerges a sufficient numerical quorum of molecules to provide the right degrees of geometrical freedom and constraint, thus exhibiting the collective electromagnetic interrelation between them that generates these effects. A primitive light-sensitive spot on the head of an amphibian, with enough accumulation of transparent focusing tissue, "amounts to" an eye. A sufficiently large collection of self-organizing nerve cells "amounts to" a mind. An over-insured and over-secure real estate market can suddenly "amount to" an economy where people owe more money than there is in the world. When a bank becomes "too big to fail" a threshold has been crossed. A competitive market can unexpectedly produce an automobile or a personal computer. Cosmos, in this sense, is the world of thresholdiness, of "amounting to," of emergence.

It is the existence of thresholds at all that is so remarkable. Why shouldn't everything in the universe increase and decrease in an orderly linear fashion, instead of—as actually happens—undergoing sudden qualitative leaps when certain thresholds are crossed? One clue might be suggested by that exponential increase in the mathematical difficulty involved in computing the outcome of a system's activity that accompanies the approach toward a threshold.

This difficulty, familiar to all who deal with limit theory and knots in mathematics, is itself a sensitive index of what we might call "thresholdiness." In sociology it is Polanyi's "unmanageability of social tasks." In economics it is the point where the relatively predictable and controllable effects of pricing in an individual firm give way to the impossible problem of state-controlled prices. Indeed, the measure of difficulty, its tendency to increase exponentially with the accumulation of variables in nonlinear systems, and its differential rates of increase in different circumstances, may be primitively constitutive of time itself, a fossil of the original instability that must have triggered the emergence of temporality. The resemblance between Claude Shannon's equations governing information (Shannon, 1948) and Boltzmann's governing the increase in entropy is very suggestive. The threshold (the present moment) of the past (all that might be known for certain) abuts upon the radical difficulty of predicting the future. All we need for there to be a future at all is non-computability.

Paradoxically, new things emerge because thresholds await them. The thresholds are both necessitated by mathematical logic and encountered in the physical world. New things are like the bucket of water perched upon the proverbial door that will descend upon the unlucky victim of the practical joke when he pushes it open. What are the conditions for that practical joke, what makes it possible, what rules would one need to have a universe free to invent radically new things without succumbing to mere inconsequential anarchy?

THE NEED FOR GRANULARITY IN A WORLD OF THRESHOLDS

One of those conditions is that a cosmos capable of threshold-crossing emergence cannot be totally dependent on continuous variables. It cannot be decomposable into more and more minute gradients of quantity, cannot be fully understood by the smooth bell-shaped curves of probability. Nor can it be controlled by a *taxis* that requires the accurate prediction of the future. Is the world made up of smooth gradients or distinct parcels, or some combination of both? The answer seems to be the last. The wave-particle argument in optics and mechanics is only one of many examples of the issue. The point here is that a "thresholdy" universe must be granular—quantized—at some fundamental level, even if at other levels it behaves in smooth analogue curves. It must be made up of "pixels," so to speak, which are atoms in the old Greek sense of the term. (Contemporary particle physics now knows of much smaller pixels than the atoms, but in the Greek sense those smaller irreducible chunks—whether quarks or strings—are the new atoms).

"Time," said Heraclitus, "is a child playing a game of draughts; the kingship is in the hands of a child" (Fragment 52), which I take to mean that the mutual predicting contest, the second-guessing that gives all games their suspense and thrill, is at the heart of the nature of time: the strange asymmetry between the past and the future; the predictable and the retrodictable; the reversible and the irreversible. The Hindus, too, regard time as a *lila*, a game. And all games require the equivalent of distinct counters, turns, and players. A tennis ball is either in or out. A chess turn, a chess piece, and a chess square are fundamental quanta (granules) of the game. Without turns, the players cannot synchronize enough to have a contest at all. Without individual players with distinct interests neither prisoner in a "prisoner's dilemma"[2] could wish to rat out his accomplice. A market requires distinct rules and counters, whether the terms of contracts and bonds, the definition of fraud, or the denominations of its currency. Electronic calculation is itself a useful game, using distinct ones and zeroes; all over the world engineers are looking for ways of making smaller and smaller secure thresholds to hold and transfer bits of information. Even quantum computers only kick the problem—of keeping the counters of the game distinct—down to the quantum level. The "calc" in "calculation" is a Greek pebble or abacus bead used in geometry and arithmetic, and also in children's board games of ancient Greek times.

Paradoxically, it is only when we play with distinct pieces, and the defined rules that identify them, that the true mysteries and discoveries can happen: because it is only if lines are sharp and definitions granular that fertile paradoxes can appear.

It does not matter if those counters and turns—the quanta and *chronons* of the world—are only relative to some particular feature of the universe, say sound or light or living cells. They can be fundamental and relative at the same time. No event can be shorter than the Planck time or happen in a smaller space than the Planck length; no sound for a human ear can be shorter than one twenty-thousandth of a second, 20 kHz being the highest pitch it can hear. No piece of light is smaller than the wavelength of its photons. No cytological activity can take place on a smaller level than a cell. No vote can be cast by less than one person; no sonnet recitation last less than about thirty seconds.

THE PROBLEMS OF AN ANALOGUE UNIVERSE

Let us perform a thought experiment and imagine a purely probabilistic world, a "straw man" to demonstrate the need for granularity. This is not to say that the "smooth" aspects of the universe—those that are quantifiable and divisible all the way down, and are subject to probabilistic expression and statistical analysis—are an illusion or unimportant or an obstacle to progress. Much of the universe, much of the time, is fairly accurately describable by approximations and averages, and we are fairly safe when we "round things off." Most butterfly wing-beats in Brazil do not create hurricanes in Florida. Many varying conditions do indeed regress to the mean. Chi-square tests for goodness of fit are rightly persuasive. But the success of probability theory as a way of predicting events and describing states too complex to be tractable in terms of Newtonian determinism, and its reliable use in the thermodynamic understanding of gases, work, entropy and even quantum mechanics, have led to an overestimation of the extent of probability's writ. Probabilistic mathematics can handle negative feedback that creates homeostasis, but not positive feedback when it crosses thresholds that define new natural states. The resulting errors are especially glaring in evolutionary biology, the social sciences, public policy, and the arts and humanities.[3] The mutation that triggers the emergence of a new species; the assassination that triggers a world war; the dream that inspires a masterpiece; events such as these cannot fit a system of standard deviations. What makes a human being a human being is precisely what differentiates her from her demographic.

The predictable world would be one of continuous gradients and variations in mixtures. Shannon (1949) points out that information can only be transmitted, and indeed exist at all, if the magnitude of its departure from the default state of its medium, channel, or carrier-wave is enough to cross some threshold that distinguishes it from noise. But the predictable world would be, so to speak, all noise. It cannot make explosions or compounds (as opposed to mixtures). It would be all bell curves devoid of cusps or catastrophes. It has no states of matter: Gibbs' free energy function,[4] which governs such phenomena as freezing, boiling, melting, evaporating, precipitating, condensing and so on, does not hold because there are no natural thresholds to cross. No new species, no new ecological niches, no new works of art could emerge into existence, crossing the boundary from the unimaginable to the possible.

If everything merges smoothly into everything else, if everything gradually becomes everything else, there can be no game. Points could not exist, and thus could not cluster together to make lines. Lines could not stitch themselves into planes. Planes could not rumple and frill themselves into volumes. Time could not have distinct beats, and thus length; it could not mount up and thus could not have a direction; it would be an eternal amorphous cloud of becoming. The change of phase among solid, liquid, gas, and plasma, between crystalline and amorphous, could not happen. If the world happened to be endowed originally with a certain amount of order and free energy, the impossibility of unique new combinations—there being no unique and bounded entities to recombine—would certainly dictate the increase of possible happenings, but it would also gradually exhaust its stock of *qualitatively different* happenings through the increase of entropy over time.

Politically and economically a predictable world might at first glance appear to be the *dirigiste's* paradise. *Taxis* would be all-powerful. All one would need to make something happen would be to apply the right amount of power and money (adding a little extra to account for the operation of the second law of thermodynamics, which would create waste heat). One could predict the results because feedback would be impossible and surprises inconceivable. Since everything would blend into everything else, there could be no polycentricity, for without boundaries there could be no centers, and without centers no sources of relatively independent causation and nodes of resistance. But the masters of *taxis* would get little satisfaction, for every exercise of their presumed power would weaken them, and the world that resulted would be diminished in its order and exhausted of its available work energy. Progress would be the mining and burning of whatever order remains, a perpetual regression to the mean, and the acceleration of the heat death of the universe.

Functional individualities—locally stable equilibria—make available the strategic back-and-forth of feedback; the competition and cooperation among regimes of crystallization or polarization in a metastable melt, among rock anemones in the ocean, predators and prey in the steppes, stock investors in the market, or cities and nations in global politics, that lead to emergent ecological niches, technologies, and polities. It is only by such interactions that things can "amount to" something other than themselves, that the whole can be greater than the parts, and that the crises, bouleversements and dénouements of evolution can be free to occur. Without distinct notes, there could be no music. Without distinct words, there could be no language. Without distinct lines, there could be no poetry.

TIME AS DIFFICULTY

The difficulty of calculating difficult algorithms, like the solution of factorials or the traveling salesman problem, is due directly to the nestedness of sub-calculations and sub-sub-calculations that must be solved before each step in the process. Out of this recalcitrance emerges a primitive form of sequentiality, an asymmetry between the ease of, for instance, the simple multiplication of a set of numbers, and the difficulty of the reverse, that is, the extraction of the factors of the large numbers that result.[5] Significantly, a quantum computer, clumsy at classical computation, can in theory solve factorial problems with ease, being unburdened by temporal order, while a classical computer, struggling with scheduling problems, is quickly stymied when the number to be factored gets too large. We might speculate that each new emergent entity in the world is the latest attempt at solving the paradox of the co-existence of both kinds of computation.

INVENTING A FREE (AND THEREFORE SURVIVABLE) UNIVERSE

If one were tasked to invent a survivable universe, that is one that still exists as does ours, it would be hard to avoid the singularly ingenious solution to the problem that we find in this one. A survivable universe is one that generates a new moment every moment, a new moment that reliably encodes the previous moment but is not encoded by it. It must be retrodictable but not fully predictable: it must be genuinely branchy as we go forward in time, and genuinely single when we look back at it. Such a universe must be continuous in both space and time (or it would not be one but many universes). But the continuity should not be trivial. It must be continuous but asymmetrical with respect to space and time.

The solution seems to be to make the basic constituents of the universe quantized; but make the logic by which they interact with each other and with themselves smoothly probabilistic. Then let its logic transform to digital once a certain size and duration threshold (the quantum/classical divide) is passed. The fine-grained logic of the universe is fuzzy; the coarse-grained logic is hard-edged and granular. The basic quanta of our hypothetical universe, its atomic pixels, work together by probabilistic rules of combination—quantum logic—rules that are different from those of its coarse-grained logic, which is classical, Aristotelian. Make the world out of very tiny indivisible pebbles, or calculi, and make them chunk—or amount to something greater than the sum of their parts—only at certain specific thresholds. But make the fine-grained logic, by which their interactions and their chunkings happen at the most fundamental level, probabilistic and always analogue and curvy, or branchy and inexact at some level of magnification.

Then let a more digital logic emerge in the interactions of the chunks that result. In large numbers those chunks themselves will still exhibit collective statistical properties, but only up to the point where some threshold of overcrowding suddenly appears, such as when enough molecules exist in a space to constitute a gas with emergent collective properties like pressure and temperature. But the really ingenious twist is that those chunks must compete for existence; and their existence, their individuation, is assured only by their internal process being so difficult to predict that they cannot be absorbed by some more complex and unpredictable chunk or system of chunks, with its own prepared niche and procedure for modeling and incorporating subordinate chunks. They are game-players already, unconsciously out-thinking each other. This evolutionary process produces structures that act *in anticipation of each others' actions*, creating a new indeterminacy of strategic competition and co-operation.

The final result of the struggle was the emergence of very large and complex individual organisms such as us: we possess the emergent property of freedom, an instantiation of the paradox of autonomy. Autonomy literally means "the making of rules for ourselves that we obey"—and the paradox occurs when we ask whether we are obeying the rules when we make them up, and whether once we are obeying the rules we have made, we are still as autonomous as when we made them up. Are we constrained to only make rules that are amendable, like the U.S. constitution? Is such a constraint itself amendable, as when we bind ourselves to a solemn promise? Such a promise may be our freest moral act—a choice not only of what we do, but who and what we are. The match between such hypothetical issues—predicted by the tension between the digital and the analogue, the probabilistic and the "thresholdy"—and our actual experience, is quite striking.

SOME SOCIO-ECONOMIC AND POLITICAL EXAMPLES OF THRESHOLD-CROSSING, AND THEIR MORAL IMPLICATIONS

A very important kind of granular individual is what Edmund Burke called "the little platoon," a human group or team of up to around 200 members—army platoons and companies, orchestras, ships' crews, faculty departments, theater companies, small businesses, sports team organizations, law firms, scientific research teams, extended families, villages, and so on. Anthropologists confirm that apparently a human being can know up to about 200 people well, and be aware of the personal obligations he or she owes to each one of them. Below that threshold gift-exchange and quasi-kinship relations are possible: above it we need markets and legal systems. Clever rhetorical and communicative devices and extreme stress can create larger artificial platoons such as tribes, racial nations on a war footing, and large cooperatives such as *Mondragon*: the danger is always that in the absence of more impersonal and automatic constraints such as the market, such organizations are in what Hobbes called a "state of nature" with respect to one another.

A second example of threshold-crossing is summed up in the expression "too big to fail." If you owe your creditor $10,000, he controls you; if you owe him $10,000,000,000, you control him. If too many borrowers are using the same security (for example the sum of world real estate, or the good faith and credit of the government), a crash is inevitable.

A third example is the city, as it has been discerningly examined by several urban theorists. Depending on the technology and the laws, there is a critical mass of population and communication at which it becomes a net exporter of creations, ideas, and new forms of organization.

There seems to be, as *Federalist 10* foresaw, another level of critical mass in population, geographical extent, and number of ethnic, religious and economic groups above which factions, instead of becoming damaging biases to a republic, cancel each other out as dangers to the public interest. They can then become useful game partners in a debate from which new solutions to problems can emerge. The need to make coalitions to be effective mitigates the exorbitance and narrowness of each faction's claims.

A fifth kind of threshold may be the emergence of a middle class larger than any other social group, say at about 38 percent of the population. Wealth suddenly increases, political freedom emerges, population stabilizes, and the likelihood of aggressive warfare diminishes.

Though emergence can be dangerous and even catastrophic, none of the good things of the world could have come about without it, and there may be an inherent tradeoff between risk and true benefit in this regard. If we are willing to accept the risk (with whatever safeguards we can contrive that will not abort the process), the following implications should follow from the analysis presented here:

1. Functional individuality, with its locally stable equilibrium, is fundamental if emergence is to be expected in the socio-political and moral arena. Since cells and organs cannot take part in social intercourse except as means, the fundamental kind of individual in this arena is the human person. Of the various individualities—the family, the little platoon, the city, the corporation, the organized religious faith, the state, the market, the international union—the human person has by many orders of magnitude the highest degree of integration and internal communication: even the internet has not yet reached the level and volume of connectivity of a single human brain. This shows the priority of the human person as the primary value and most important element in ensuring the continued production of emergent order, and the basis of human *cosmos*.

2. Difficulty is the sign of potential emergent order. Difficult times and "hard cases" indicate the point where experiment should be exercised—for instance, by a robust federalism and the encouragement of different sets of rules among the states and municipalities. Putting the human person in a much smaller and less powerful local arena can unleash its potential for the discovery of new "attractors" such as the American novel, the airplane, the Hudson School of landscape painting, the transistor, and the social website.

3. New order may look deceptively simple, because old tangles, knots, and problem complexes are now accepted as players in the game rather than as obstacles to be straightened out. Partisan politics always resists new order because it cannot conceive of its enemy as a valuable debating partner. This point is a subtle one and really needs an essay of its own.

4. Time the destroyer is time the creator. Be prepared to wait while a critical mass of some unexpected parameter is accumulated.

5. Emergence creates new territory. Theories of economic justice that try to find some basic stuff (like land, or "social capital," or even, absurdly, genetic inherited talent) to apportion out fairly among persons fail to recognize that emergence changes what is the basic stuff itself. Tiny medieval cities like Genoa, Venice, Amalfi, and Amsterdam became world empires in the entirely new world space of international markets. The airplane turned the air into "land." Banks made capital into a sort of fundamental "stuff." Radio bandwidth did the same thing with the cosmic electromagnetic field. Cyberspace became wildly valuable real estate. There are no "natural resources" in the sense accepted by such groups as the Club of Rome; the crossing of a new threshold creates new natural resources. The combustion engine transformed poisonous goo in the ground into fuel; the silicon chip made sand a natural resource.

A CONCLUDING REFLECTION

Chaos theory and complexity theory have supplied us with mathematics, models, and images for re-conceptualizing the world in an emergentist perspective. Much work has already been done in this direction:

> What the new science has done in effect is to place within our grasp a set of very powerful intellectual tools—concepts to think with. We can use them well or badly, but they are free of many of the limitations of our traditional armory. With them we can dissolve old procrustean oppositions—between the ordered and the random, for instance—and in the process reinstate useful old ideas like freedom. New concepts, such as emergence, become thinkable, and new methods, such as nonlinear computer modeling, suggest themselves as legitimate modes of study. I have divided these new conceptual tools into six categories: a new view of cause and prediction, a richer understanding of feedback and iteration, a revolution in the idea of time, an anthology of new recognizable structures and shapes, the idea of the attractor as a way of dissolving old dualisms, and the technique of modeling (Turner, 1997, pp. xi-xxvii).

If we are troubled by the reflection that as rational thinking beings we can have no intuitive understanding of the process of emergence, or that it is pointless to try to analyze the inherently unpredictable, there is a talent that we possess that may console us. I have already hinted in this essay that the creation of art and poetry may be good examples of emergence and threshold-crossing. In the essay "Beauty and the Anima Mundi," Turner (1991) proposes that the human aesthetic sense is precisely the capacity that an advanced animal with brain tissue to spare might develop to both guess and contribute to the course of emergence as it occurs around us on both the large and the small scale. What we find beautiful may be said to be what is about to emerge, what is emerging, what reveals its emergence. Art is the way that we use the hugely complex, multiply iterative, and astonishingly adaptive tissue of our nervous system to continue the invention of the world. A new work of art, whether a sonata, a fresco, a sonnet, a scientific theory, or a lovingly raised child, is the most improbable thing in the world. and the most valuable for that reason.

NOTES

1 A Peano curve is a fractal curve (not differentiable) that, although consisting of a simple line, fills the whole plane between determined x,y limits.

2 Two men are arrested, but the police do not have enough information for a conviction. The police separate the two men, and offer both the same deal: if one testifies against his partner (defects/betrays), and the other remains silent (cooperates with/assists his partner), the betrayer goes free and the one that remains silent gets a one-year sentence. If both remain silent, both are sentenced to only one month in jail on a minor charge. If each ”rats out” the other, each receives a three-month sentence. Each prisoner must choose either to betray or remain silent; the decision of each is kept secret from his partner. What should they do? If it is assumed that each player is only concerned with lessening his own time in jail, the game becomes a non-zero sum game where the two players may either assist or betray the other. The sole concern of each prisoner seems to be increasing his own reward. The interesting symmetry of this problem is that the optimal decision for each is to betray the other, even though they would both be better off if they cooperated.

3 Rémy Lestienne (personal communication) points out that Ludwig Boltzmann, like Josiah Gibbs one of the founders of thermodynamics, never believed that nature was "analogic" and thought that "analogic" physics was only a mathematical trick. Lestienne adds that Henri Bergson went so far as to identify true freedom of will as itself exclusively a moment of creative emergence—"*les actes libres sont rares*"—while most of our decisions remain physiologically determined.

4 See http://scienceworld.wolfram.com/physics/GibbsFreeEnergy.html and http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/helmholtz.html#c2 for useful definitions.

5 Michael Heller's (2003) *Creative Tension: Essays on Science and Religion* argues that noncommutativity, i.e. an asymmetry in logical order in respect of the identity relation, is all that is needed to get a universe. Turner (2011) writes: "For Heller, the essential issue is how timeless mathematics—which, he argues, miraculously does truly describe the real world—actualizes itself in matter and time. He suggests interestingly that the mathematics of quantum theory is non-commutative. That is, unlike commutative mathematics in which 3×7 is the same as 7×3, the non-commutative mathematics of the first moment of the Big Bang and of any space in the present universe smaller than the Planck length dictates a difference in the state of a system according to the order in which a mathematical or logical operation is performed. That is, if 3×7 is not the same as 7×3, the difference is the fundamental unit of space and time. Given such units, a whole universe can evolve without outside assistance through the now familiar processes of selection, self-organization, and emergence in nonlinear dynamical systems. . ."

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