Models of Human Action

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Abstract: Standard modeling procedures treat the behavior of economic variables using stochastic-determinate equations. Two alternatives to this approach arose in the last quarter of the twentieth century: deterministic nonlinear equation models and agent-based models. The class of deterministic nonlinear equation models produce dynamic behavior without exogenous shocks, but these models are populated by agents incapable of self-organizing. Agent-based models, on the other hand, allow us to study interactions between synthetic actors and the phenomena that emerge from these interactions. In so far as economic phenomena—like prices, firms, nation states and business cycles—are products of human action but not human design, agent-based models are a more productive way forward in formal modeling of complex adaptive systems such as a market economy.

Keywords: Complexity, Agent-based Models, Hayek, Self-organizing Systems.

> The distinction between simplicity and complexity raises considerable philosophical difficulties when applied to statements. But there seems to exist a fairly easy and adequate way to measure the degree of complexity of different kinds of abstract patterns. The minimum number of elements of which an instance of the pattern must consist in order to exhibit all the characteristic attributes of the class of patterns in question appears to provide an unambiguous criterion
> (Hayek 1964, p. 333).

I. INTRODUCTION

Prices, firms, nation states and business cycles are some prominent phenomena of modern economic systems. Standard economic analysis begins with the presumption that these phenomena are *simple,* not *complex* (Foster 2005). Consider the following treatment of "prices" or "exchange ratios". Robinson Crusoe is on an island by himself. He has preferences, endowments and production possibilities.1 As a worker he sells labor and earns wages, as a firm he buys labor and produces goods, as a consumer he buys goods. He exchanges with himself, and the trade-offs he makes are represented as "exchanges ratios" and are called "prices". In this conception, prices are a *simple* phenomena because the minimum number of individuals necessary for their emergence is one. As Buchanan (1964) stressed, such isolated action easily shifts attention toward optimization and allocation, and away from the striving for evolution toward a solution that constitutes the ongoing exchange process in a complex adaptive economic system.

Real world prices are very different from prices in a Crusoe economy. Prices are a system of telecommunication that allows individuals to coordinate their plans. The system of prices along with other institutions allow us to communicate and coordinate, so significant is this idea that an economic system may be defined as a "set of roles tied together with channels of communication" (Boulding 1956, p. 205). Prices emerge out of the interaction between many individuals, some of whom are buyers, others sellers, yet others arbitrageurs. These interactions are often guided by social norms and structured through institutions like stock exchanges. Robinson Crusoe does not need a system of telecommunications because there is no one to communicate with. He can create coordination between his plans as a worker, a firm and a consumer simply by ratiocination. In the real world, prices emerge when an economic system becomes so complex that other mechanisms of coordination fail. The characteristic attributes of the price system—-like stock exchanges, financial instruments and banking—-arise only at a certain scale. Prices are a complex phenomena in a Hayekian sense because the minimum number of elements necessary for the emergence of all its characteristic attributes is fairly large.[2]

However, the question of scale that relates to the price system is not scale in a statistical sense. For instance, the law of large numbers is a consequence of scale when individual elements are *independent*. In complex phenomena, scale matters because individual elements are interdependent and self-organizing. In a large society with identical individuals with identical and unchanging goals, coordination problems do not become more complex as the number of individuals increases. Scale matters in human systems precisely because individuals pursue their own goals and the market as such has no teleology. Prices serve to guide exchange and production decisions in a world with a multiplicity of ends and a scarcity of means. The relevant knowledge must continually be discovered by the participants in the system. The price system is a consequence of human action, but not human design. Individuals form interdependent plans and engage in exchange, these exchanges generate prices. No individual intends to create the collection of attributes we call the price system, nor does any individual posses these attributes in isolation from others, nonetheless it is their actions that creates this system. Once the price system emerges, it guides and motivates individual actions. The relation between the parts and the whole involves many channels of feedbacks (Wagner 2010). Individual action is autonomous but it is influenced by the nature of the system within which they act.

Individual-level actions generate system-level dynamics. Stock prices, GDP, composition of goods, price level, number and size of firms, all change over time. And the changes in these aggregate variables in turn influence individual-level actions through the guiding role of relative prices, and the discipline of profit and loss statements. Within contemporary formal theory there are three ways to model the dynamics of an economic system: stochastic-determinate models (SDM), non-linear deterministic models (NDM), and agent-based models (ABM). Each of these techniques corresponds to a different vision of human action and the economic system.

SDM have two elements: a set of linear difference or differential equations describing economic actors and a set of exogenous shocks. Linear systems are not capable of producing reasonable economic dynamics without exogenous shocks. This caricature of human beings goes back to psychological theories of the first half of twentieth century, from behaviorism to existentialism. Bertalanffy (1968) called this the 'robot view' of human beings. Humans are thought of as little more than impulse-response systems. Ragner Frisch, Robert Lucas and others developed the idea by modeling a collection of human beings as a single impulse-response system (Lou 1997). Lucas does not believe that each individual behaves like the representative individual, but that a system with a large number of agents behaves like a representative individual. He believes that the behavioral characteristics of individual economic actors wash out in large systems, so that the system as a whole behaves like a single rational optimizing agent (Hoover 2013). This view assumes away the interdependence between the plans of individual economic actors.

NDM use nonlinear difference and differential equations to describe individual economic actors. These equations are capable of producing dynamics without exogenous shocks. In essence, NDM replace the well-behaved robots of linear models with mutant avatars. Robots of linear models walk in straight lines, occasionally thrown off by exogenous shocks, only to return to their ordained path. The mutant robots of NDM walk along all kinds of strange paths and have a penchant for sharp turns. However, the robots that populate the SDM or NDM are incapable of self-organizing. They cannot discover prices, language, money, firms, churches and nation states. Neither SDM, nor NDM view an economy as a self-organizing system.

In contrast, ABM allow us to study the interaction between actors and the phenomena that emerge from these interactions. ABM are synthetic economies *in silico*. They are populated by agents who have motives, cognitive abilities and behaviors, and they interact within a defined rule environment. In some ABM, agents form rules using meta-rules (Luke 2009). Prices, firms and other economic phenomena emerge out of these interactions. In contrast, economic actors of SDM and NDM do not meet each other, they meet vectors of equilibrium prices. ABM are built on the idea that human systems possess a capacity to self-organize; as to how poorly and well they do so will be a function of the rule environment which governs their social intercourse.

The paper is organized as follows. Section 2 and 3 discuss SDM and NDM respectively. Section 4 discusses ABM and compares it to other modeling techniques. Section 5 offers concluding remarks.

II. THE QUEST FOR STOCHASTIC DETERMINISM

Ragnar Frisch (1933) in a pioneering paper claimed that the problem of economic dynamics ought to be divided into two parts: *propagation problem* and *impulse problem*. Frisch modeled an economy as a static system that is periodically hit by external shocks. The "shocks are absorbed in a continuous fashion by the system which acts as a stable resonator" (Frisch 1939, p. 639). The *impulse problem* is concerned with the distribution from which external shocks arrive. The *propagation problem* is concerned with the properties of an economic system that determine how it responds to external shocks. The existence of internal mechanisms that amplify or dampen external shocks means that there need not be any synchronism between an external shock and its impact on an economic system (Frisch 1933, p. 1). Short-lived shocks may produce responses with a long lifetime and shocks of small magnitude may elicit relatively large responses. Lucas (1975), for instance, presents a model where the effect of monetary and fiscal policy are distributed over time because no agent has perfect information about the state of the economy. Bernanke and Gertler (1995) present a model in which small policy shocks have significant impact on output and employment through the credit channel.

Frisch's bifurcation by itself was not novel. Such a bifurcation was a part of the analytical schema of Hume (1752), Wicksell (1898), Mises (1912) and Hayek (1933) who analyzed the propagation problems that arise as a consequence of monetary shocks. Frisch's ingenuity lies in having transformed Walras's static system into a dynamic model, without giving up its determinism:[3]

> In one respect, however, must the dynamic system be similar to the Walrasian: it must be determinate. That is to say, the theory must contain just as many equations as there are unknowns. Only by elaborating a theory that is determinate in this sense can we explain how one situation grows out of the foregoing. This, too, is a fact that has frequently been overlooked in business cycle analysis. Often the business cycle theorists have tried to do something which is equivalent to determining the evolution of a certain number of variables from a number of conditions that is smaller than the number of these variables. It would not be difficult to indicate example of this from the literature on business cycles (Frisch 1933, p. 2).

Frisch stood in sharp contrast to Hume, Wicksell, Mises and Hayek, all of whom analyzed out-of-equilibrium dynamics that arise due to monetary shocks. Hayek (1945, p. 91) thought that the obsession with equilibrium modeling led economists to "habitually disregard an essential imperfection of man's knowledge and the consequent need for a process by which knowledge is constantly communicated and acquired". In contrast to the intellectual framework of modern technical economic analysis, this Hayekian perspective, begins with a fundamental indeterminism: problems and variables are known but the equations are unknown. Demand and supply curves are not given, rather they reflect market relations that are discovered by economic actors (Hayek 1948). In the post-war years, much of the economic profession chose Frisch's stochastic-determinism over Hayek's fundamental indeterminism. So much so that "the 1960s and 1970s witnessed an almost complete unanimity on the use of linear-stochastic models in order to understand business cycles" (Boldrin 1988, p. 1).

III. DETERMINISTIC-NONLINEAR DYNAMICS MODELS

Unlike SDM, DNM work with *non-linear* difference and differential equations**.** Linear difference and differential equations are capable of generating one of four possible time paths. A time series may converge or diverge, and it may do so with or without oscillations. Divergence with oscillations means oscillations of ever increasing amplitude. Convergence with oscillations means that oscillations die out. Neither convergence nor divergence with oscillations reflect time-series economic data. SDM get around this problem by introducing exogenous shocks.

In contrast, nonlinear difference and differential equations are capable of producing dynamics which combine convergence, divergence, oscillations of various amplitudes and sharp turns that are characteristic of the onset of the economic crisis;[4] all this without exogenous shocks. Such systems are capable of chaotic behavior, whose "central characteristic is that the system does not repeat its past behavior" because of sensitivity to initial conditions (Baker 1996, p. 1). Richard Goodwin was one of the first economists to work with nonlinear equations (Venkatachalam and Velupillai 2011):

> By dropping the highly restrictive assumptions of linearity we neatly escape the rather embarrassing special conclusions which follow. Thus, whether we are deal-

> ing with difference or differential equations, so long as they are linear, they either explode or die away with the consequent disappearance of the cycle or the society. One may hope to avoid this unpleasant dilemma by choosing that case (as with the frictionless pendulum) just in between. Such a way out is helpful in the classroom, but it is nothing more than a mathematical abstraction. Therefore economists will be led, as natural scientists have been led, to seek in nonlinearities an explanation of the maintenance of oscillation (Goodwin 1951, pp. 1-2).

However Goodwin worked with models which did not have microfoundations. The nonlinear equations were written on Keynesian statistical aggregates or Marxian classes (Goodwin 1967). This made Goodwin's approach vulnerable to the rationality critique: if economic agents maximize quasi-concave utility functions, then why would they not arbitrage away non-linear oscillations? After all, they prefer smooth consumption to variable consumption. In the 1980s, Jess Benhabib, Richard Day and Jean-Michel Grandmont among others set about to answer this critique.[5] They developed models with optimizing rational individuals that produced nonlinear dynamics.[6]

Consider an over-lapping generations (OLG) model. Each generation—depicted by a representative agent—maximizes its utility function given constraints defined over state variables. The outcome of this optimization problem, combined with market equilibrium conditions, yields a relationship between state variables today and tomorrow. A system with two state variables, x and y, will yield equations of the following form: xt+1 = f(xt, yt) and yt+1 = g(xt, yt).

Typically, structures are imposed on the preferences and production function so that the functions *f* and *g* are linear. Non-transversality conditions are imposed to ensure that the state variables converge to a steady state on a saddle path. Such linear systems have been used to study a variety of economic phenomena including growth (Solow 1956), pensions (Azariadis 1993) and government bonds (Barro 1974).

However without restrictive assumptions on preferences and production functions, nothing guarantees that the functions *f* and *g* will be linear. This opens the doors to a wide variety of dynamics, including sequences that do not converge to a steady state value or well-behaved cycles. Benhabib and Day (1981) show that an OLG model can produce erratic behavior if preferences depend on realized past consumption. Benhabib and Day (1982, p. 37) "characterize and give examples of wide classes of utility functions which generate erratic dynamics in the standard, deterministic, overlapping generations model".

The one area where erratic behavior is most evident is economic growth. The United States has experienced a steady growth rate for more than two centuries, but "the US experience is the exception rather than the rule. Much of the world is characterized by miracles and disasters, by changing long-run growth rates, and not by countries with stable long-run growth rates" (Easterly and Levine 2002, p. 4). The tranquil world of neoclassical growth models does not come close to the experience of large swatches of humanity. Day (1982, 1983) shows that NDM yield chaotic dynamics, and arguably a better description of the world. Not too implausible assumptions on utility functions are capable of generating nonlinear dynamics—even chaotic behavior—in models with optimizing individuals.[7]

IV. AGENT-BASED MODELS

ABM is a technical means to study an economy as a complex adaptive system. In such systems, "economic agents (firms, consumers, investors) constantly change their actions and strategies in response to the outcome they mutually create" (Arthur 2013). And this interaction between economic actors creates a ecology of related plans (Wagner 2012). Some of these plans are mutually compatible, others are mutually incompatible, some relations are symbiotic, others are parasitic. A nexus of relations arises as a consequence of human action and in turn influences human action. This view of an economic system is fundamentally different from an equilibrium view, in which economic actors interact with an array of prices but do not interact with each other.

ABM are fundamentally different from SDM and NDM because they are not solved using as many equations as unknown. This motivates why they do not have the word 'deterministic' as a post-fix. An ABM is an synthetic economy *in silico*. It contains synthetic agents and rules of interaction among these agents. Each agent is a bundle of behavioral rules and an information set[8] (Axtell 2007). Agents may be 'intelligent', heuristic decision makers or random actors (Gode and Sunder 1993; Cliff and Bruten 1997). Similarly, agents may have access to local or global information. ABM are 'solved' by running the system forward in time and analyzing the resulting data. Which is why Axelrod (2007) says ABM is a "third way of doing science":

> Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theo-

> rems. Instead, a simulation generates data that can be analyzed inductively. Unlike typical induction, however, the simulated data comes from a rigorously specified set of rules rather than direct measurement of the real world. While induction can be used to find patterns in data, and deduction can be used to find consequences of assumptions, simulation modeling can be used as an aid intuition (Axelrod 2007, pp. 92-93).

Schelling (1971) built one of the first ABM.[9] Schelling analyzed segregation in US cities along racial lines. His basic intuition was that segregation is not necessarily an outcome of discriminatory preferences of a vast majority of individuals. In other words, macro-level properties are not necessarily possessed by micro-level actors. Rather macro-level properties may be an emergent outcome of individual choices. He placed two types of agents on a checkerboard. Each type had a mild preference for living in a neighborhood with some agents of the other type, but did not wish to live in neighborhood where the majority were of the other type. Initially agents were randomly distributed over the board. Each period agents are allowed to move to a nearby location which they prefer to their current location. Overtime one sees the emergence of spatial segregation despite the mild preference for diversity.[10] The Schelling Model is an example of an invisible hand explanation.[11] Social phenomena emerges out of the interaction between purposive actors who interact within an institutional setting. The emergent phenomena may be socially desirable (like market prices that guide economic activity) or undesirable (like racial segregation). The normative leaning of the analysis is institutionally contingent.[12]

ABM is one way, perhaps the best contemporary way, to pursue methodological individualistic approach to capture complex phenomena. It emphasizes exchange relations that emerge between economic actors pursuing their own ends.[13] Economics is neither the study of Robinson Crusoe on an island by himself, nor of a competitive equilibrium (Buchanan 1964). In a Crusoe economy, there are no economic problems to solve. In a competitive equilibrium, there are economic problems but economic actors do not solve them. In Buchanan's (1964, p. 218) words:

> A market is not competitive by assumption or by construction. A market becomes competitive, and competitive rules come to be established as institutions emerge to place limits on individual behavior patterns. It is this becoming process, brought about by the continuous pressure of human behavior in exchange, that is the central part of our discipline…

The institutions that emerge through the process of competition may be viewed as "knowledge". In the same way that knowledge in the human brain is stored in the strength and nature of *connections* between individual neurons, knowledge in a market economy is stored in the nexus of relations between economic actors (Hayek 1952; Smith 1997). A nexus of relation emerges as economic actors pursue their own ends given the existing structure. Potts (2001) draws the distinction between 'markets as an information processing system' and 'markets as a knowledge creating & organizing system'. The mechanism design literature studies markets as information processing systems given a set of relations between economic actors, i.e. given a state of knowledge. ABM provide an opportunity to study markets as knowledge creating & organizing systems, albeit an under-exploited opportunity. Consider Miller's (2001) study of the evolution of organizations using genetic algorithms. He models organizations as a collection of individuals and relations between them. A number of organizations with random relations between individuals is initialized. Over time organizations evolve through a process akin to natural selection. After several time periods, the fitness of organizations increases and all kinds of interesting structures emerge within organizations. These structures can be thought of as a *knowledge* embedded within an organization. They determine how the agents interact within an organization.

4.1 ABM and Stochasticity

The relationship between stochasticity and ABM is multifaceted. ABM use random number generators, however they work very differently from SDM. Stochasticity enters ABM in three ways. First, agents may be initialized with random draws of parameter values, for instance Axtell (2002) uses random draws from a uniform (0,1) distribution for Cobb-Douglas exponents in the utility functions of agents when they are initialized. *Second*, at the individual level random numbers may be used to model unpredictable behavior or forces that are exogenous to the model. Suppose, two agents meet, one of whom is willing to pay $10 for an apple and the other willing to accept $5. Presumably, the price at which they trade will depend on their relative bargaining power. If the origin and consequence of bargaining power are exogenous to the model, one can assume that they trade at a random price between $5 and $10. *Third*, random numbers may be used to instantiate the rules of interaction. Imagine a model where agents meet through a process of binary matching. There are a variety of ways to implement such an algorithm; one way is to select two agents from the population using a random number generator. This process is known as a random activation.[14]

While in SDM, the law of large numbers washes out the stochastic elements to produce thin-tails, nothing guarantees such *thinning* in ABM. The reason is that in ABM individual level shocks work their way to system level outcomes through the interaction between agents. These interactions are capable of transforming shocks from well-behaved distributions into fat-tails of state variables. In fact, a basic rule of thumb in ABM is to use draws from a uniform distribution so that most—if not all—of the properties of the distribution of state variables is due to interaction between agents rather than random number generators. SDM too are capable of transforming shocks, however they are incapable bringing about as radical a transformation as ABM (Axtell 2014).

Consider the size distribution of firms in the US. It resembles a Pareto distribution for which the first moment is barely meaningful and the second moment does not exist. The modal firm has one employee, the mean firm has 19 employees. (Axtell 1999). However neither mode, nor mean are representative statistics. Walmart for instance employs nearly 1% of the US labor force[15]. US firm size distribution has fat tails. It would be difficult to build a SDM that would produce fat tails in firm size distribution. This can be done with an ABM. Axtell (2002) presents an ABM that produces over twenty stylized facts about US firm size distribution including fat tails.

Frisch was right in saying that there need not be a 'tight' relation between initiating force and the response of the system, however in SDM the relationship cannot be too 'loose' either. This is because SDM rule out interaction between economic actors. One illustration of the difference between a tight and a loose relation is the contrast between Lucas's (1975) Islands Model and Mises's Cycle Theory. In Lucas's Island Model, monetary disturbances create real miscoordination by driving a wedge between the perceived relative price and the actual relative price. This happens because agents observe the price level with a delay. In Mises's Cycle Theory, monetary disturbances create real miscoordiantion by driving a wedge between the market rate of interest and the natural rate of interest. In Lucas's model, there is a monotonic and predictable relation between the size of the monetary shock and the consequent real miscoordination. No such relation exists in Mises's Cycle Theory; the size, scope and nature of the real miscoordination depends on numerous factors like which agents get the newly create money, what they do with it and how the money travels around the economy system. The loose relation between the initiating force and the response of the system in Mises Cycle theory is due agent-interactions that happen as a consequence of the initiating force. In Lucas's Island, agents do not interact with each other, they interact with an array of prices. This makes all the difference.

4.2 ABM and Nonlinearity

Like DNM, ABM are capable of producing nonlinear dynamics.[16] Traditionally, the dynamics between prey and predator population have been studied using the Lotka-Volterra equations. These are systems of nonlinear equations that produce rich dynamics which are sensitive to initial conditions. ABM too can be used to study these dynamics. For instance, Wilensky's (2005) presents a ABM of predator-prey dynamics. Wilensky's ABM, like the Lotka-Volterra equations, is capable of generating a variety of dynamics including the extinction of both species and cyclical movement of the two populations. It works as follows:

> "…wolves and sheep wander randomly around the landscape, while the wolves look for sheep to prey on. Each step costs the wolves energy, and they must eat sheep in order to replenish their energy—when they run out of energy they die. To allow the population to continue, each wolf or sheep has a fixed probability of reproducing at each time step" (Wilensky 2005).

Though both ABM and DNM produce nonlinear dynamics, they reflect two different views on the *complexity.*[17] In DNM 'complexity' means erratic behavior of data.[18] We shall call this the 'output view of complexity'. In ABM 'complexity' means structural features of a system. A complex system is one in which *interactions* between parts of the system can generate properties at the system-level which were neither possessed by the components parts, nor were easy to deduce from the properties of component parts (Simon 1962). We shall call this the 'structural view of complexity'.

Lloyd (2001) lists more than twenty different definitions of complexity. He divides these definitions into three categories. The first category is concerned with how difficult it is to describe a system. The second category is concerned with how difficult it is to create a system, for instance computational complexity measures the difficulty associated with creating solutions to well-defined problems. The third category is concerned with the degree of organization of a system. The DNM definition of complexity belongs to the second category of Lloyd's taxonomy; it can be computationally difficult to solve systems of non-linear difference equations. The ABM definition of complexity belongs to the third category of Lloyd's taxonomy; the structure of relations between economic actors matters. Interestingly, both Simon (1962) and Hayek's (1967) views on complexity falls with the third category of Lloyd's taxonomy. Hayek (1967) claimed that the structures that emerge from the interactions between agents depend on the number of agents that interact. For instance, an economy consisting of Robinson Crusoe and Friday on an island is unlikely to use prices as a system of communication, though one might be able to impute exchange ratios. If we endow Crusoe and Friday with preferences used in Benhabib and Day's OLG models, the Crusoe-Friday economy will produce erratic behavior. However we would not call the Crusoe-Friday economy 'complex' in the Hayekian sense because only two elements were necessary to produce these patterns. Similarly, a miswired clock can produce nonlinear dynamics, but the clock is not a 'complex' system in the Hayekian sense. The global economy however is a complex system because many of its 'characteristic attributes'—like prices, stock markets, fat-tailed distributions wealth, firm size and nation states—may not exist at the scale of a ten or hundred people.[19]

V. CONCLUDING REMARKS

"The experience with which the sciences of human action have to deal is always an experience of *complex* phenomena" (Mises 1998, p. 31, italics added). Complex phenomena—like prices, social norms and formal institutions—emerge out of the interaction between purposefully acting human beings.[20] As Hayek (1974) noted in his Nobel lecture, an economy is a self-organizing complex system, where the properties of the macro-structures depends no only on the properties of the individual economic actors but also on how they connect with each other. How economic actors connect with each other is not predictable, rather it depends on how human beings respond to circumstances (Buchanan and Vanberg 1991). Human beings are heterogeneous not just in preferences and endowments, but in how they creatively respond to problems posed to them:

> A stone is a thing that reacts in a definite way. Men react to the same stimuli in different ways, and the same man at different instants of time may react in ways different from his previous or later conduct. It is impossible to group men into classes whose members always react in the same way (Mises 1957, p. 5).

SDM view economic actors as impulse-response systems. And NDM view economic actors are mutant robots. Economic actors in neither model are capable of human action, and the system as a whole is not capable of self-organizing. Like characters of Beckett's play, they wait for a Walrasian auctioneer to compute prices and tell them what to do. Such a view of economic actors cannot be a useful foundation to understand emergent phenomena that are a consequence of human action but not human design. Agent-based models are a better way forward in understanding the emergence of such phenomena. Agents-based models are the technical counterpart to complex methodological individualism. Both recognize the relation between the parts and whole, and the variety of feedbacks between the two levels.

NOTES

1 A variety of standard models attempt to study systems with multiple individuals; these include general equilibrium theory, search theory and game theory. However, these models are not fundamentally different from the model of a Crusoe economy. Fundamentally, the standard approach seeks to derive *fixed points* from which the economy would not deviate. In this vision, prices are parameters that define the fixed point. For instance, general equilibrium theory works with many agents, some of whom are producers and others consumers. Equilibrium is defined as an arrangement in which consumers maximize utility, producers maximize profits and excess demand is zero. Both the Crusoe economy and general equilibrium theory avoid the question of price discovery. The Crusoe economy avoids the question because there is a single agent. General equilibrium theory avoids the question by assuming that forces outside the system compute prices. Search-theory, which an extension of general equilibrium theory to a stochastic environment shares much with the Crusoe economy (High 1983). In search-theory the distribution of equilibrium prices is computed by an authority outside the system. Neither the Crusoe economy, nor general equilibrium theory or search theory ask how price emerge from the interaction between economic agents. Similarly, Nash's theorem establishes conditions under which non-cooperative games have fixed points. No questions are asked

about how a system of interacting agents might discover such fixed points, if at all.

2 Pareto (1906) made the observation that when an economy becomes too large, it becomes infeasible to solve systems of equation to determine general equilibrium, even if they are given. He said that a decentralized system of autonomous interactions may solve such a problem faster than a centralized system that attempts to solve equations. Once an economy is large enough, political economy would come to the aid of mathematics. Recent results on the computational complexity of Brouwer's fixed-point theorem suggest that Pareto's comments were indeed insightful. Brouwer's fixed point lies in the class-NP, these are problems for which no known polynomial time algorithm exists, i.e. they cannot be solved in a reasonable time by a computer. Axtell (2005) shows that decentralized processes with autonomous agents can solve *some economic problems in a polynomial time.*

3 For a history of stochastic models in economics see Mirowski (1989).

4 For a survey of various nonlinear dynamics see Baumol and Benhabib (1989).

5 For a survey of deterministic-nonlinear dynamics models up to late 1980s see Boldrin (1988) and Lorenz (1993). For a discussion of the difference between deterministic-chaos and stochastic models from a statistical point of view see Bartlett (1990).

6 They developed some models of the old Keynesian fashion too. Day and Shafer (1985) show the emergence of chaotic dynamics in an IS-LM style model.

7 For a survey of more recent application of nonlinear dynamics to economics see Barnett et al (2015).

8 This ties in very nicely with the idea of encapsulation in Object-Oriented Programming.

9 Schelling did not build the first ABM. In fact, James M. Sakoda built a model of segregation with autonomous agents on a grid nearly three decades before Schelling. See Hegselmann (2014) for a discussion on why James M. Sakoda is an unknown pioneer. Also, see Vriend (1999) for a discussion on Hayek as a proto agent-based computational economist.

10 Wilensky (1997) built a Netlogo version of Schelling's model. There are some technical difference between Schelling's model and the Netlogo version. Schelling's model had sequential activation, i.e. agents moved sequentially beginning from those in the top left hand corner. The Netlogo model has uniform activation, i.e. all agents get to move every period. Another difference is that in Schelling model agents move to the nearest "satisfactory" square with distance measured in terms of squares one traverses horizontally and vertically. Whereas in the Netlogo implementation one moves to the most preferred plot in the vicinity, i.e. only one step at a time.

11 See Nozick (1974, pp. 18-22) for an explanation of what constitutes an "invisible hand explanation" and a list of what he considered at the time to be exemplars of "invisible hand explanations".

12 Eaton and Lispey (1975) built one of the first ABM of an economic system. They wished to study Hotelling-type spatial competition in a two dimensional space with more than three firms; a problem which is "very complex, perhaps intractable using conventional techniques" (Eaton and Lispey 1975, p. 40). In the years since, ABM have found a variety of applications in economics (Amman et al. 2006) and finance (LeBaron 2000, 2006) including asset pricing (Arthur et al. 1997), evolution of cooperation (Axelrod 1984; Axelrod 1997), industrial organization (Axtell et al. 2002; Axtell 1999), retirement behavior (Axtell and Epstein 1999), and emergence of states and nations (Cederman 1997).

13 Boettke (2012) makes a distinction between "mainstream economics" and "mainline economics". Mainstream economics studies optimizing rational agents making decisions in an institutionally antiseptic environment. Mainline economics, on the other hand, studies the emergence of institutions that facilitate exchange and competition between individuals pursuing their own ends. Mainline economics is all about squaring the rational choice postulate with the invisible hand theorem via institutional analysis. Also, see Simpson (2013) for a discussion of the difference between classical and neoclassical economics in their treatment of institutions, emergence and complexity.

14 Other activation regimes include uniform activation and sequential activation. In uniform activation agents are ordered in a list and then selected one by one. In sequential activation agents are selected in a particular sequence. In Schelling original model agents on the top left hand corner as the first to move and those on bottom right are the last, this is an example of sequential activation. Activation regimes may have importance consequences on model outcomes, see Nowak and May (1992), Huberman and Glance (1993) and Axtell (2001).

15 According to the World Bank US labor force in 2014 was 158,686,472. According to Walmart, in the US it employs more than 1.3 million associates.

16 This is not to say that every DNM can be represented using an ABM and vice-versa. This is an area where theorems are hard to come by.

17 See Rosser (2009) for a review of a wide variety of work that goes under the label of "complexity economics".

18 "This paper uses the familiar, neoclassical theory of capital accumulation to show how *complex* behavior can emerge from quite simple economic structures. Indeed, when sufficient nonlinearities and a production lag are present, the interaction alone of the propensity to save and the productivity of capital can lead to growth cycles that exhibit a wandering, sawtooth pattern not unlike those observed in reality. These fluctuations need not converge to a cycle of any regular periodicity so they are not quasi periodic" (Day 1982, p. 406, italics added).

19 In recent years, macroeconomists have begun studying the structural features of economic systems, with the hope that these may shed light on the behavior of macroeconomic variables. Hidalgo and Hausmann (2009) study the structural properties of international trade networks to understand the process of economic growth. Acemoglu et al (2012) and Carvalho (2014) study how the structure of production networks affects the evolution of macroeconomic variables.

20 Boettke (1989) explains how the "Austrian School" has a long tradition of using evolutionary ideas for economic analysis. And in this sense the Austrian School has much in common with institutionalists. Both emphasize change and the influence of institutions in guiding human action.

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