Preferential Attachment and Carl Menger's Theory of the Endogenous Emergence of a Medium of Exchange

CASEY PENDER

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Abstract: This paper outlines Carl Menger's theory of monetary evolution through a network science perspective. Drawing on the works of Price (1976) and Barabasi and Albert (1999) among others, parallels are drawn between Menger's account of a tradable good emerging as one half of every exchange and the mechanisms by which certain papers rise to the top of a citation list or how certain websites are vastly more popular than others. This phenomenon of spontaneous self-organization into a hub-and-spoke network is known as preferential attachment. I argue that by viewing the endogenous emergence of money as a preferential attachment process, monetary economics could be enriched. Finally, this paper highlights the spontaneous self-organization of complex trade networks leading to the creation of a medium of exchange as evidence to bolster the classical liberal perspective while cautioning against attempts to build such complex networks from the top down.

Keywords: Networks, Preferential Attachment, Menger, Endogenous Money

1. INTRODUCTION

Language, webpages, brains, social arrangements, and ecosystems. Traditionally, these are topics studied in different departments and in different groups with little overlap. Yet in the past quarter century, researchers have begun to see commonalities in the way in which systems such as these operate—general trends, and even rules, which seem to dictate the evolution of networks independent of the make-up of the network components. The study of this has become known as network science: an interdisciplinary endeavor aiming to understand the complex connections observed in many different academic fields (and in everyday life). Physicists, neuroscientists, computer scientists, biologists, epidemiologists, and even the occasional economist all study networks and have begun to see and share these commonalities that bridge their respective disciplines.

Perhaps one of the most interesting and fruitful discoveries of network science is that as many real-world networks evolve, they spontaneously self-organize into what can be characterized by a hub-and-spoke system. A hub in this context is a node which is vastly more connected to the network than other nodes. For example, Google is a site that links to far more websites than the average website, New York City is bigger than the average American city, and *The Use of Knowledge in Society* by Hayek (1945) is cited by far more economists than the average economic paper. Studying these hubs is crucial to understanding the networks in which they exist. Due to the way they dominate a network, nearly all travel through the network will pass through a hub, and therefore, hubs have the ability to massively influence the system. According to Barabási and Bonabeau (2003, p. 62), finding hubs in so many seemingly unrelated and different types of networks offers a "convincing proof that various complex systems have a strict architecture, ruled by fundamental laws—laws that appear to apply equally to cells, computers, languages and society."

Though writing nearly a century before network science became a distinct discipline, when Carl Menger (1950 [1871], 1892) gave an account of how media of exchange emerge through trade, he seems to have anticipated some of the key features of this strict architecture. Menger describes a hub-generating feedback loop where successfully traded goods beget success in future trades. In network science this phenomenon is now known as "preferential attachment" (Barabási and Albert 1999). In the context of Menger, this preferential attachment continues until one good eventually becomes so widely accepted that it is one half of every exchange. What we call money, then, is whatever good evolves into the hub of the trade network.

The goal of this paper is to explore Menger's theory of the endogenous emergence of money as understood through the lens of modern network theory, in particular using the concept of preferential attachment.[1] It is my conjecture that through this new connection, fruitful avenues for collaboration may open which could aid in the further advancement of Mengerian research program. Not only would this present an opportunity to better understand why we use money and its central role in trade, but it also has implications for studying spontaneous orders and classical liberalism more generally.

Section 2 of this paper gives an overview of some of network science's more recent history while going over some important concepts and terminology. This section also presents a simple and general model of a growing network with preferential attachment. It is hoped that by seeing the exciting work being done in this field, readers will be drawn to comparisons with economics and see how the tools of network science can help further our research.

Section 3 then presents Menger's theory of the endogenous emergence of money through the lens of modern network theory, seeing trade as a pattern of human interaction. Thus, it is shown how as money begins to come about, it sets off the feedback mechanisms of preferential attachment, which spontaneously organize the economy into a hub-and-spoke network. This section further discusses some of the more recent endogenous money models and their relationships with Menger; specifically, those working within Monetary Search as well as more modern work directly in the Mengerian tradition, or what Salter and Luther (2014) have termed the "Spontaneous Order" tradition.

Section 4 begins by asking if the resulting network structures from spontaneous processes are likely to be more desirable outcomes than consciously designed networks. It is argued that by traders having incentives to be interested in a good's properties beyond just acceptability, first-mover advantage and other worries can potentially be overridden. Using more of the recent network science literature, it is also cautioned that imposing a hub-and-spoke trade network on a group of traders from the top down might be too difficult of a task to be successful.

Finally, section 4 ends with a discussion of network attack and failure. While hub-and-spoke networks are shown to be incredibly robust to random node failure, they can be quite frangible if the hub is disrupted. In this sense, while the spontaneous pattern of monetary trade which emerges from preferential attachment is a very resilient system as a whole, it can go awry in the face of monetary disruption. This is an important area of further study because it is all too easy to confuse the effects of money mismanagement with market failure. If the underlying network topology is not well known and understood, misguided and illiberal policies can be the result. Section 5 offers concluding remarks.

2. BUILDING NETWORKS THROUGH PREFERENTIAL ATTACHMENT

The beginning of network science as a unique field of study arguably begins with the works of Erdős and Rényi (1960) who, according to Barabási (2016), were the first to combine probability theory, combinatorics, and graph theory for the study of networks.

Before explaining these early network models, it may be helpful to first think of what is known as a complete network—a network in which each component, or node, is connected with every other node via a link. A relevant example of a complete network is a Walrasian auction house. In such a place every imaginable good can trade for any other good (at the right price) instantaneously. Menger (1892, p. 243), however, believed this was not an accurate depiction of real networks of trade:

> It is an error in economics…that all commodities, at a definite point of time and in a given market, may be assumed to stand to each other in a definite relation of exchange, in other words, may be mutually exchanged in definite quantities at will.

Indeed, most networks are not complete, and so modeling them as such can result in erroneous understanding and prediction (Amaral et al 2000). No airline has a direct flight to and from every airport in the world, I do not know or interact with all the billions of other humans on the planet, and a single neuron in your brain only shares a synaptic connection with a fraction of your other neurons. Real networks are most often sparse, meaning that the average amount of links one would have to travel along to get from one node to another is greater than one. How sparse a network is, then, can be quantified by the average path length between the nodes in the network.[2]

Recognizing this common attribute of sparsity in networks, Erdős and Rényi (1960) began to model nodes as connecting to each other randomly. They constructed an elegant model where they envisioned the probability of two nodes connecting as determined by a Poisson distribution, making the network topology essentially random. While the specific shape of each Erdős and Rényi network shape is unpredictable *a priori*, general rules or patterns can be studied and understood. For example, the degree of any node—the number of connections between it and all other nodes—will likely not differ significantly for the average degree.[3]

Yet, as mentioned in the introduction, many networks have hub nodes—nodes that dominate the network in terms of connectivity. Though the Erdős and Rényi model was truly ground-breaking, the existence of these types of outliers, or network hubs, cannot be explained using Poisson distributions or any other random connection mechanism.

One of the first proposed models of an underlying process that could explain the existence of hubs came from Price (1976).[4] Price began by observing that heavily cited academic papers are more likely to be cited again in the future; there is a positive feedback loop where success begets success. As Price (p. 292) generalized it:

> A paper which has been cited many times is more likely to be cited again than one which has been little cited. An author of many papers is more likely to publish again than one who has been less prolific…Words become common or remain rare. A millionaire gets extra income faster and easier than a beggar.

Motivated by these observations, he modeled the most recent academic papers as being newly created nodes which enter the network and preferentially attach to older papers.[5] A link between nodes thus represents a citation, and a node's degree tells how many citations that paper has had. With this set-up, his model was able to predict hubs—papers that have degrees far above the average degree, or in other words, get cited vastly more than most other papers.

Price (ibid.) understood that his methods could be extended beyond citations and lamented that the "full elegance" of modeling these types of feedback loops was not yet "widely appreciated." It was not until the later internet boom that researchers outside of bibliometrics and information sciences took network evolution more seriously (Newman 2018).

When Barabási and Albert (1999) presented an algorithm for producing these hub networks in a simple and generalizable way, things really began to take off. Unlike Price's model where links between nodes were directional, Barabási and Albert thought of connections as having no direction, or alternatively could be thought of as going in both directions. While the dictional model makes sense for citations, Barabási and Albert's set-up is a better model for trade because trade, by definition, goes both ways.[6] From here they build a network with two simple rules: "(i) networks expand continuously by the addition of new [nodes], and (ii) new [nodes] attach preferentially to sites that are already well connected" (p. 509). With only two rules, they find that the resulting topologies match real-world networks better than the Erdős and Rényi model—like Price's model, their preferential attachment model creates hubs.

Further details and extensions of preferential attachment models have been numerous (see Krapivsky and Redner (2001), Dorohovtsev et al (2000), Bianconi and Barabási (2001) for examples), and have been used to model evolving networks in fields as diverse as linguistics (Dorohovtsev and Mendes 2001) to ecological webs (Williams and Martinez 2000). Drawing from this body of work, we can outline a simple model of a growing network exhibiting preferential attachment which can be used in the next section to better understand Mengerian trade networks and money in this context.

Let us begin with X randomly connected nodes at time t_0,[7] where we assume that a new node enters the network at each discrete time interval, such that the total nodes, $N(t)$, in the network at any given time, t, is equal to $X + t$. Whereas the initial X nodes are connected randomly, each new node i that enters at $t = t_0 + i$ connects preferentially to $c \leq X$ existing nodes, much like as in Barabási and Albert (1999). This can be stated as $\rho_{i,j} \propto \alpha_j + q_j(t)$, which says that the probability of new node i connecting to existing node j is proportional to both the fitness of j, represented by α_j, and the amount of connections node j already has at time $t_0 + i$ is represented by $q_j(t)$.[8] While α_j is a positive constant, $q_j(t)$ can vary with time as existing nodes can gain new links. For simplicity, if it is further assumed that new nodes entering the network always create a single link to an existing node, i.e. $c = 1$ and $\sum_{j=1}^{N} \rho_{i,j} = 1$, then, in its simplest form, our model of preferential attachment can be summarized by:

$$\rho_{i,j} = \frac{\alpha_j + q_j(t)}{\sum_{k=1}^{N}(\alpha_k + q_k(t))} \qquad \text{(equation 1)}$$

This would surely need to be built upon to fully capture all of the network properties found in Menger and which would make up a comprehensive model exhibiting an endogenous emergence of money. For example, building in mechanisms for the addition and subtraction of links between existing nodes, such as in Dorogovtsev and Mendes (2001), would be a good start. Nonetheless, a simple setup as presented here in (1) offers a "minimal proof-of-principle model whose main purpose is to capture the basic mechanisms responsible for the emergence" of hubs (Barabási 2016, p. 192).

With a mechanism for preferential attachment established, the next section shows how this can be applied to Menger's endogenous money theory, and how such formulation fits within other monetary scholarship.

3. MENGER'S ENDOGENOUS MONEY: A MODERN PERSPECTIVE

While there are exceptions, some of which will be discussed later, the workhorse models in monetary economics currently fall under the paradigm of Dynamic Stochastic General Equilibrium (DSGE). While these techniques have allowed for greater understanding of the effects of central bank policy, they make no claims as to why agents would require a medium of exchange to conduct trade. In fact, many influential DSGE models (following Woodford (2003)) do not have money as a variable at all.

Some effort has been made to bring money directly into DSGE models. For example, using the cash-in-advance restraint of Clower (1967) has been a clever tactic to bring money into the utility function and thus have a medium of exchange play a role in DSGE (see Chari and Kehoe (1999) and Danthine and Kurmann (2004) for some examples). Yet, even here, these models exogenously impose that money must be used in trade and have little to say about why or how it is that people come to use a medium of exchange in the first place.

This tension arguably stems from the Walrasian general equilibrium framework on which much of monetary economics is based.[9] Thinking in terms of networks, assuming any good can trade for any other good is akin to a complete network where the average path length between any two nodes is one. With such a topology, no unique hubs exist, and there is, therefore, nothing unique about money in terms of its connectivity to other goods.

In his writings on money, Menger (1950 [1871], 1892) works to establish a much different framework. His story of the emergence of money begins in a world of barter, however, Menger does not conceptualize this initial bartering stage as a complete network (like a Walrasian auction), but a sparse network where few trades can take place on a regular basis. Thinking of classes or groupings of goods as nodes (where links between nodes signify the possibility of trade between these goods) and mapping out the topology of this initial trade network, we would expect a relatively small number of nodes, with the trade network exhibiting high average path length.[10] Referencing back to the previous section, the nodes in this initial barter phase could be modeled as X at t_0.

Furthermore, the background conditions for Menger's endogenous money model do not contain rational utility-maximizing agents with full information who are able to probabilistically form expectations on future events as in so many models common today. Instead, Menger envisioned people as he saw them acting in the real world: as fallible but capable actors trying to do their best with what they have.[11] Understanding this, it becomes apparent that traveling between nodes incurs a cost. Given a non-zero cost from trading, or "traveling", through the network, it is conceivable that this could outweigh the benefits of trade, making many otherwise mutually beneficial trades prohibitively costly. Thus, the initially high average path length implies "the number of bargains actually concluded must lie within very narrow limits" (Menger 1892, p. 242). In this setting, with the dark forces of time and ignorance abounding, trading with others is risky business.

Some agents may take on this risk, however, if they believe the good they are receiving in trade is likely to be more generally and readily acceptable in the marketplace than the goods currently held (Menger 1950 [1871], p. 260). The entrepreneurial innovation of these risk takers then is the understanding that there can be longer paths between nodes which can be traveled via indirect trade. If successful, this strategy will allow its implementers to eventually obtain more of the goods and services they ultimately desire compared to those who only to engage in direct barter alone. According to Menger, once this first entrepreneurial step has taken place, the remaining agents will eventually adjust their heuristics in order to attempt to mimic these successful indirect trading strategies, for there is "no better way in which men can become enlightened about their economic interests than by observation of the economic success of those who employ the correct means of achieving their ends" (1950 [1871], p. 261). This can be thought of as the means by which preferential attachment enters the narrative. General acceptability, or the degree, of good i is equivalent to the total subjective value of the good as influenced by the $q_i(t)$ term in (1),[12] which Menger (1892, p. 248 emphasis in original) clearly sees as influencing the decisions of traders:

> [W]hen any one has brought goods not highly saleable to market, the idea uppermost in his mind is to exchange them, not only for such as he happens to be in need of, but, if this cannot be effected directly, for other goods also, which, while he did not want them himself, were nevertheless more

> saleable than his own. By so doing he certainly does not attain at once the final object of his trafficking, to wit, the acquisition of goods needful to *himself*. Yet he draws nearer to that object.

Thus, as traders continually seek out goods which are highly connected there is "an increasing differentiation" between highly connected goods "and that of all other goods" as a higher q begets even higher q's (Menger 1892, p. 252). As such, as the system approaches a stage where $q_h(t)$, the hub's degree, approaches N, the system as a whole will approach an average path length of two. If each movement between nodes is costly in and of itself as we have assumed, then $q_h(t) \rightarrow 2$ implies a drastic reduction in transaction costs from sparce barter networks or other more random networks of indirect trade. And so "without any agreement, without legislative compulsion, and even without regard to the public interest," a hub-and-spoke network forms which allows for far greater levels of mutual beneficial trades (Menger 1950 [1871], p. 260).[13]

Many of the key insights in this story begin with Menger's break from the starting assumption that trade networks are complete. While much of current monetary economics still holds this complete network assumption implicitly (or explicitly) within their GE framework, not all monetary work has. Many have continued to research in the Mengerian spirit and start with the assumption that trade networks are sparse. Starting with Jones (1976) and then Kiyotaki and Wright (1989, 1993) and Wright (1995) to name a few, those working within Monetary Search theory have followed this line of reasoning and have done much to advance our understanding of money. Though the network terminology was not yet available to him, Jones (1976), for example, clearly understood the special role of hubs in the trade network. More recently, Luther (2014, 2016b) has made strides to link insights from Monetary Search with insights from Menger.[14]

While these search models capture important aspects of sparse networks, they do not fully incorporate the all of dynamics of growing networks where nodes attach preferentially. The early Kiyotaki and Wright models did not have new goods continually added (the network did not grow) and so there was no opportunity for preferential attachment. Furthermore, the workhorse model of Monetary Search theory developed by Lagos and Wright (2005) is a two-stage model where Walrasian markets operate at one of the stages. As beneficial to our understanding of money as search theory has been, it is still a sub-field fully entrenched in equilibrium-based economics.[15] It is mainly concerned with end states/steady states—if agents within the economy ultimately engage in indirect trade or if they do not—and is somewhat less concerned with modeling the process by which the use of money (or failure to do so) begins to come about. It is within this process where network theory can be applied. As Lavoie et al (1990) put it, "what is of interest is not what the end results are so much as how the process works."

For Menger, the spontaneous societal convergence on a medium of exchange is not instantaneous; instead, it is an evolutionary process with feedback loops slowly molding the shape of the network. This feature in Menger has also been recognized as a general feature of networks, as Bianconi and Barabasi (2001, p. 436) confirm:

> A generic property of these complex systems is that they constantly evolve in time. This implies that the underlying networks are not static, but continuously change through the addition and/or removal of new nodes and links.

Network theory in this regard seems to bring to the forefront process oriented and open-ended systems for which Menger had seen a need for in the study of human interaction and trade, but which is otherwise often downplayed or outright ignored.

Those working within what has become known as the Spontaneous Order tradition (Salter and Luther 2014) are, not surprisingly, the greatest exception to this, and have written extensively on the processes, particularly the feedback loops, guiding the evolution of hub-and-spoke networks. White (1984, 2002), for example, clearly discusses the feedback loops necessary in Menger's theory and Selgin (2011 [1988], p. 80) even refers to money as a "hub." Of those who have formally modeled endogenous money emergence in the Mengerian spirit, Klein and Selgin (2002), Howett and Clower (2000), and Luther (2016a, 2018, 2019) perhaps come the closest to doing so from a network science perspective as advocated for here.[16] Klein and Selgin (2002) use a Polya urn process—an early version of preferential attachment—to depict the feedback loop leading all traders to eventually accept money. Howett and Clower (2000) borrow from biology and take an evolutionary model with similar feedback loops. Luther (2018, 2019) divides up the value of goods into two components that match up well with (1) presented here. These papers in particular seem to be getting at the preferential attachment element present within Menger's writings on money. While they have developed Mengerian models with modern techniques, they appear to be doing so unaware of, and independent from, the large body of network scientists who are modeling similar networks in other fields. It may be possible that taking their models and building on them by borrowing from existing models within network science could open new doors of discovery within monetary economics.

4. NETWORK DESIGN AND NODE FAILURE: IMPLICATIONS FOR LIBERALISM

If the arguments thus far are correct, then bolstering the Mengerian research project with network science will illuminate how a medium of exchange *can* spontaneously emerge. However, this on its own does not speak to whether it *should* spontaneously emerge. One possible concern is that while preferential attachment can ensure a money emerges and reduces the cost of trade relative to barter and other more random indirect trade patterns, there is no guarantee that the good which ends up as money is in some sense the socially optimal choice. In other words, what if there are other goods out there that could be a better medium of exchange for society as a whole under certain important criteria, but which lose out due to first-mover network effects? After all, there are other seemingly inefficient institutions that persist in the real world. On this, White (2002, p. 271 emphasis in original) notes that "[s]ocial scientists have indeed not yet discovered any *universal* tendency towards better social institutions, any single mechanism that yields superior institutions *in all cases*."

Preferential attachment in and of itself is not sufficient to produce efficient outcomes. Putting this in network science terms, setting $\alpha_i = 0 \; \forall \; i$, essentially eliminates the fitness of a good from affecting its probability of connecting with other nodes and eventually becoming a hub. In this case, all that affects the probability of future connections is past connections. Like in Barabási and Albert (1999) and in Klein and Selgin (2002), without fitness being modeled, first-mover advantage becomes a key driver in what becomes a hub and certain nodes deemed less desirable than other nodes can become "locked in" as hubs (Arthur 1989). When modeled as such, essentially there is no intrinsic properties within a node which can help predict its future degree. This could be problematic when evaluating networks from a social welfare perspective as researchers may have good reasons for thinking there are intrinsic properties which *should* influence the network topology. For example, towards the end of the nineteenth century many thought Esperanto was a much more logical and efficient language than the most commonly spoken European languages (Janton 1993) yet it never became wildly adopted. As another example, the US government in the 1970's declared "that the metric system is the preferred system for weights and measures in both trade and commerce of the United States" (Buchanan and Chang 1997) though its attempts towards its adoption were unsuccessful.

As already well documented by White (2002), however, given the logic in Menger's explanation of the endogenous emergence of money, it is in fact not very likely that the condition $\alpha_i = 0 \; \forall \; i$ would ever hold in a forming network of trade. This is because people must take into account things like portability and durability etc.—in other words the goods' monetary fitness —if they are able to successfully navigate indirect trade.

White (2002) points this out with an example of an entrepreneur looking to engage in indirect trade in the early stages of the Mengerian story. In this case the agent must understand the important physical properties of the prospective good to accept in order to understand the likelihood of being able to trade this good again at a potentially unknown time and unknown place. If the agent plans to use a single good acquired now for many future trades, then they will likely be concerned about divisibility. Similar type considerations would apply to "cognizability, durability, and stability of purchasing power." White (2002, p. 277 emphasis in original) continues:

> *Whatever* are the properties money must have to better satisfy the preferences of its users, the users of pre-monetary exchange media have a private incentive to seek those properties in choosing among exchange media.

While this paper is only meant to draw the connection between Menger and modern network theory, it is hoped that future research will be able to build networks based on equations like (1) which can experiment with different values of αi relative to $q_i(t)$ to see under what conditions the nodes which are deemed the most fit have the best probabilities of becoming hubs. Further additions of elements such as link deletions could help study the environments in which switching from one medium of exchange to another and under could be possible. What is for certain is that successful indirect trade would require agents taking the fitness of the goods which they are accepting into account,[17] and as such, modeling the trade network as suggested here can (at least in part) eliminate worries of hubs forming regardless of important intrinsic features of the node itself.

Thus far, it has been suggested that a properly specified preferential attachment model could result in the spontaneous emergence of a socially optimal medium of exchange. Yet this on its own does not mean it is necessary for a socially optimal medium of exchange to arise in such a manner. In some circumstances, consciously planned and designed networks operate quite well. Take FedEx, which has its shipping hub in Memphis, meaning that most goods flow through there before their final destination (O'Kelly 2015). That did not come about spontaneously by FedEx employees slowly and spontaneously converging on Memphis. Instead, it was a conscious decision from management to arrange their distribution network as a hub-and-spoke system. In fact, Cancho and Solé (2003) have shown that when designing networks, having the designer work through an optimization problem will often result in the same hub-and-spoke topology as if it had evolved via preferential attachment.

And so, providing a solution for evolving networks says little about their preferability over designed and consciously planned networks with similar topologies such as FedEx's distribution network. However, while it is true that we can observe hub-and-spoke networks that are consciously planned, this seems to be the exception rather than rule. As Newman (2018) notes, hub-and-spoke networks are most often the "result of a succession of random processes, often decentralized and quite blind to the large-scale structure they are creating."

Furthermore, setting up such a system in a firm, like FedEx, is not the same as setting up such a system by a government. While FedEx employees are incentivized to follow the commands of their employers, getting citizens to follow government commands in a similar context may be a much more difficult task. While organizing a firm within society may be difficult for those in charge, it is still an order of magnitude less complex than the networks connecting society as a whole.

For example, Dorogovtsev and Mendes (2001) have studied language as a prime example of an evolving network which is not amenable to conscious design: Language is "so complex that it cannot be controlled but rather organizes itself while growing" (p. 2603). This principle extends beyond language to many other social institutions, as Hayek (1973, p. 24 emphasis added) observed: "The theory of complex spontaneously formed structures with which social theory has to deal, can be understood *only* as the result of a process of evolution." Likewise, Barabási (2016, p. 190) asserts that in the formation of almost all complex networks, "structure and evolution are inseparable."

Menger (1982, p. 250) did not think a planned hub-and-spoke network could be successfully implemented by any government from the top down:

> It is not impossible for media of exchange, serving as they do the commonweal in the most emphatic sense of the word, to be instituted also by way of legislation, like other social institutions. But this is neither the only, nor the primary mode in which money has taken its origin.

It may be that Menger, recognizing the immense complexity of the trade network, thought preferential attachment type reasoning was the only way to fully understand the endogenous origins of money precisely because such complex networks are not easily controllable. For economists to study trade from a network perspective is to see the inherent difficulty, or even impossibility, of attempts to impose arrangements and structures of the patterns of trade exogenously. This includes attempts to create new media of exchange *ex nihilo*.

In Menger's view, however, just because social planners cannot arrange the topology of the trade network themselves from the top down, does not mean that the government could have no role in money creation. Menger (1892, p. 255) believed that "state recognition and state regulation" of media of exchange could help along an evolving trade network. A government recognizing and committing to accept a particular good, *i*, as well as ensuring quality, deterring fraud, *etc*., would be equivalent to a positive exogenous shock to α_i—increasing *i*'s fitness and thus increasing the probability of it evolving into a hub. Salter and Luther (2014) have elaborated on this point using similar reasoning.

How the network evolves to have a hub is one thing, but what happens to the hub and its influence on the network afterward is another. Again, network theory can help. For starters, it can help us model when fiat money will be widely accepted and when it will not. A fiat money can be thought of as good *j* with $\alpha_j = 0$, yet where $q_j(t)$ is sufficiently high to call *j* a hub. It is clear that, under the assumptions of Sections 1 and 2, no good could become a medium of exchange if $\alpha_j + q_j = 0$ at the time *j* enters the network. Because the probability of any new node attaching *j* in this case would be zero, its chances of evolving into a hub are also zero. However, models could be constructed with an exogenous negative shock to α_j (representing a government, say, removing its currency from the gold standard), at a time in which $q_j(t) > 0$, to study the effect of transitioning from commodity monies to fiat.[18] Luther (2016a, 2018, 2019) has also used similar such models to discuss the possibility of wide acceptance of cryptocurrencies as media of exchange.

Perhaps where network theory can most help advance monetary theory for classical liberals is the study of network attacks and node failure. What Albert et al (2000) have shown is that hub-and-spoke networks are incredibly robust to random attacks or failure. By growing networks in a similar method to that of (1), they then begin to randomly remove nodes to simulate node failure. Because hubs will be so rare, a node chosen at random will almost certainly not be a hub, and as they explain further: "The removal of these 'small' nodes does not alter the path structure of the remaining nodes, and thus has no impact on the overall network topology" (Albert et al 2000, p. 380). What this speaks to in terms of trade networks is that we should expect them to be incredibly robust. Endogenously emerging money in Mengerian theory not only allows trade and economic conditions to improve, but it also forms a network that is resilient to random failure. Studying this aspect of Menger may lead to better arguments against those who portray the macroeconomy as inherently fragile or unstable.

On the other hand, Albert et al (2000) and others (O'Kelly 2015) have noted that there is an inherent fragility in hub-and-spoke networks when the removal/alternation of nodes is non-random. When hubs are deliberately targeted in this type of network, it drastically decreases the overall connectivity of the network and increases the average path length. Nodes that were once connected via a hub may get cut off completely from each other and the rest of the network, leading to a complete alteration of the network's topology.

This insight aligns well with the works of monetary economists who have long seen most depressions and network-wide failures stemming first and foremost from monetary issues (for example, Yeager (1956)). Building on their work by modeling monetary disruptions as hub failure or attack may help us better understand the trade cycle and economy-wide depressions.

As an analogy to another hub-and-spoke network, a storm at the Dallas/Fort Worth Airport can cause a ripple effect of flight delays in multiple cities across the world. Yet passengers delayed outside of Dallas are likely to understand the hub-and-spoke nature of flight networks and so can comprehend that their delays are not caused by the sunny weather at the city they are in, but in the hub airport far away.[19]

Yet the topology of the trade network may not currently be as transparent to agents using it. A disruption to the usual functioning of a medium of exchange, like a storm in Dallas, can have cascading effects in multiple other markets. However, the buyers and sellers of, say, apples may not fully understand the way in which their frustrated plans are caused by monetary disturbances and falsely blame the apple market itself.

Historically, this type of misdiagnosis has troubling outcomes for anyone sympathetic to a broadly liberal society. As Simons (1936) noted, even if bad monetary policy is the culprit for slow economic growth or outright depression/recession, it is often hard for the public to see the connection and assumes a failure of the central bank is instead a market failure. And as Sumner (2012, p. 20) has also pointed out, some of the most intrusive government interventions in history have come at a time of deep recession; the Great Depression in the USA in the 1930s precipitated many unproductive, or even destructive, government policies. The depression, even worse in 1930s Germany, may have also contributed to the ascension of the Nazis to power. More recently, in the 2000s, Argentina's deep recession led the new government to blame their troubles not on monetary issue "but on its former free-market policies, just as FDR had done 70 years earlier". Similar trends of the increased size of government can be found across the world since the 2008 crisis. Understanding the trade network's topology, then, and how hub failure can affect it, may not only help us avoid unnecessary and harmful depressions, but it could also help avoid harmful illiberal policies which may follow.

All of this is to say that hub failures are not always clear to the public, nor is the solution always agreed upon by economists or politicians. While networks evolving from preferential attachment have shown to be incredibly robust in general, they can be fragile when there are disruptions at the hub. From a classical liberal perspective, getting money right and ensuring smooth travel through the hub may be the best strategy for convincing the public of the merits of open and voluntary trade, and the social benefits of market mechanisms.

5. CONCLUDING REMARKS

Leon Walras—one of Menger's fellow revolutionaries in marginal analysis—clearly had an agenda to make economics more like Newtonian physics. He collaborated with a professor of mechanics to "conceive of the state of the market as a general problem of static equilibrium described by a system of equations" (Jaffe 1976) and asserted the "perfect similarity" in the "equations of general equilibrium with the equations of universal gravity" (Walras as quoted in Ingrao and Israel (2015 [1990], pp. 84-86). Likewise, Jevons (1892, p. 760)—another pioneer in marginal analysis—describes economics as "a kind of physical astronomy investigating the mutual perturbations of individuals."

As has been documented by Jaffe (1976, p. 520), Menger approached economics from a widely different perspective than these two—he thought in terms of "bounded indeterminacies" and saw markets as dynamic rather than static. While Menger too was influenced by the sciences, he was more influenced by ecology or biology (Beccho 2014). Yet it was the mathematical elegance of Walras and Jevons that caught the imaginations of modern economists, leaving the less-mathematical Mengerians on the outside looking in.

As Lavoie et al (1990) have argued, however, those working in the Mengerian tradition have long noted that "the mathematics of differential calculus that has played such a central role in mainstream economics is not the appropriate mathematics for studying the economy" and that perhaps the more computational techniques employed in biology and other fields looking at complex open-ended systems:

> might prove to be the kind of modeling approach that is process-oriented enough to help rather than obstruct economic theorizing. Thus it could constitute a useful new complement to the traditional procedures of theorizing that market process economists now employ.

Similar calls to Lavoie et al (1990) have been made by Hayek (1994 [1968]), Arthur (1989, 2015), Boettke and Veetil (2016), and Wagner (2020), all pointing out that many of the models and techniques used in different areas of natural science today may be more helpful for understanding human interaction than the physics that inspired Walras and Jevons. What I have attempted to argue in this paper is that network science has produced a plethora of particularly useful process-oriented modeling techniques that could help economic theorizing in understanding the role of money. This thinking in terms of evolving networks allows us to incorporate complex relationships such as feedback loops and open-ended systems without equilibria.

Furthermore, by not explicitly addressing the topology of the trade network, much of monetary economics is challenged from the outset to understand the influence that hubs can have on the system. Barabási (2016, p. 223) claims that if "we want to understand the structure of a network, we must first get its dynamics right." If Barabási is correct, then starting from Menger's assumption of sparse networks seems the better path for understanding trade networks. Getting the dynamics right is of vast importance, for, as has also been stressed in this paper, when the dynamics are modeled wrong, calls for illiberal policies can arise.

Given the complexities involved, formally modeling monetary economies in a way that is true to Menger and captures his most important insights is a daunting task. However, it could be made easier by borrowing from other fields that have already done the mathematical/computational leg work. Succeeding in this goal will give us a deeper understanding of the important role money plays in trade.

NOTES

1 In this paper I refer to money in any model of society where agents naturally and spontaneously converge on a medium of exchange as "endogenously emerging". This is not to be confused with a separate class of models sometimes called "endogenous money" models, where the use of a medium of exchange is imposed on agents exogenously, but where the total money supply is determined endogenously. "Endogenous money" in the latter sense then, is not concerned with how agents come to use money in the first place and thus not aimed a answering the same set of questions which are discussed in this paper.

2 The average path length of a complete network is one, as moving between any two nodes in a complete network requires traveling along only one link. One way to define sparse networks then is any network with an average path length greater than one. Using the citation network as an example, if a link is defined two papers is defined as one paper citing the other, then it has been estimated that the average path length between any two published works is just over eleven (Barabási 2016).

3 See Barabási (2016) for a more detailed history of the influence Erdős and Rényi have had on network science.

4 Another early study of the important role hubs play in certain networks was from Simon (1955).

5 Price used the term "cumulative advantage" to denote these types of feedback loops. The term "preferential attachment" did not enter the network lexicon until Barabási and Albert (1999).

6 Another difference between the two models is that Price (1976) imagined that the number of connections each new node made from entering the network was drawn from a random distribution. This was meant to represent the fact that bibliographies are not all the same length. Barabási and Albert (1999), by contrast, imagine that every node makes the same number of connections to the network denoted by c. For simplicity, I set $c = 1$ for all discussions in this section. Yet none of this alters the key result at hand: the generation of hubs.

7 It turns out that the size of X, provided $X < \infty$, as well as the configuration of their connections, is of no influence on the asymptotic properties of the network as $t \to \infty$ (Newman 2018).

8 No such α_i is present in Barabási and Albert (1999), therefore in their original model, the probability of attachment is solely driven by the degree of existing nodes. Under such assumptions, a node with no connections can never gain any new connections. There is also a path dependence in which the older a node is, the more connections it is likely to have. Dorogovtsev et al (2000, p. 4633) introduce an α which they call the node's "initial attractiveness" because it allows nodes beginning with no connections the possibility of obtaining connections in the

future. This initial attractiveness is the same for all nodes in the network, however. It was Bianconi and Barabási (2001) who introduced the subscript to alpha, allowing each node its own value, which they called the node's "fitness." This allows the probability of a good becoming a hub to be driven by more than just its degree and, among other things, diminishes the first-mover advantage. As will be detailed in Section 3, this αi is key to understanding Menger's theory of endogenous money as a preferential attachment process.

9 For more on the issue of having a medium of exchange within a Walrasian GE framework, see Selgin (1994) and Luther (2016b).

10 More than just having a large average path length, it seems reasonable to imagine that in Mengers initial world of Barter as having an infinite average path length. This would imply for some pairs of goods, there exists no sequence of trades which could connect the two.

11 For a more detailed exposition of how Menger viewed and modeled economic agents see Campagnolo (2016).

12 The actual term Menger used for a good's connectivity or degree was *Absatzfähigkeit*, which has been translated in Menger (1892) as the good's "saleableness". Luther (2018) sets up a similar model, referring to good *i's* degree, q_i, its "monetary value" and its fitness, α_i, its "non-monetary" value.

13 For other modern retellings of the Mengerian theory of the endogenous emergence of money, see O'Driscoll (1985) and White (2002).

14 Luther (2014, 2016b) is specifically concerned with linking Monetary Search to the monetary economics of Ludwig Von Mises. However, as Mises was heavily influenced by Menger, Luther indirectly attempts to link Monetary Search with the Mengerian concept of the endogenous emergence of money to which Mises largely subscribed.

15 While early Monetary Search papers such as Jones (1976) and Kiyotaki and Wright (1989) directly reference Menger, Menger's works seem to have fallen out of the purview of more recent works in this field. The most widely used textbook on Monetary Search, Nosal and Rocheteau (2017), does not cite Menger once (nor does it cite Barabási and Albert (1999) or any other of the preferential attachment literature).

16 While not using the type of formal models discussed here, Duffy and Ochs (1999) could be added to this list. Instead of modeling endogenous money mathematically, they were the first to use experimental economics to observe these feedback loops directly, which is an interesting and promising technique for further examining preferential attachment.

17 Menger seems to have believed that due to some of their physical properties, precious metals would have the highest monetary fitness and thus would be most likely to become media of exchange. However, there is some tension between Menger (1950 [1871]) and Menger (1892) on this. In the later work, Menger seems to be suggesting that precious metals will almost always become a dominant medium of exchange, whereas in the former work he seems less willing to predict which particular goods will become hubs. This can be somewhat reconciled, however, if we simply interpret Menger (1892) as increasing his estimates of the α of metals compared to most other goods. And so, metals under this new estimate would have a much better chance at becoming a hub, though it is still not guaranteed.

18 A similar model has been put forth by Selgin (2002) who also notes that, historically, the only successful fiat currencies have been ones which were originally pegged to commodities such as gold (or to other currencies which pegged to commodities). Thus, Selgin has shown that media of exchange can only become fiat once their hub position has already been established.

19 Rowe (2009) deserves credit for both making analogies between airports and money, as well as also noting hub-and-spoke topology of the trade network.

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